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# Some new modifications of Kibria's and Dorugade's methods: An application to Turkish GDP data

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**Abstract** In multiple linear regression analysis, multicollinearity is an important problem. Ridge regression is one of the most commonly used methods to overcome this problem. There are many proposed ridge parameters in the literature. In this paper, we propose some new modifications to choose the ridge parameter. A Monte Carlo simulation is used to evaluate parameters. Also, biases of the estimators are considered. The mean squared error is used to compare the performance of the proposed estimators with others in the literature. According to the results, all the proposed estimators are superior to ordinary least squared estimator (OLS).


## 1. Introduction

Consider the following standard linear regression model

$$Y = X\beta + \varepsilon \tag{1.1}$$

where $Y$ is an $n \times 1$ vector of dependent variable, $X$ is a design matrix of order $n \times p$ where $p$ is the number of explanatory variables, $\beta$ is a $p \times 1$ vector of coefficients and $\varepsilon$ is the error vector of order $n \times 1$ distributed as $N(0, \sigma^2 I_n)$. Ordinary least squared (OLS) method is the most common method of estimating $\beta$ and the OLS estimator of $\beta$ is given as follows

$$\hat{\beta} = (X'X)^{-1}X'Y \tag{1.2}$$

In some situations, the matrix $X'X$ has almost zero eigenvalues meaning the explanatory variables are correlated. This leads to a large variance and so large mean squared error (MSE). Thus one may not reach a reliable solution for $\beta$. This is the commonly faced problem called multicollinearity. There are various methods to solve this problem. The ridge regression is one of the most popular methods proposed by Hoerl and Kennard (1970a,b).

In ridge regression, adding a small positive number $k(k > 0)$ called ridge parameter to the diagonal elements of the matrix $X'X$, we obtain the following ridge estimator

$$\hat{\beta}_{RR} = (X'X + kI_p)^{-1}X'Y, \quad k > 0 \tag{1.3}$$

The MSEs of the OLS estimator and the ridge estimator $\hat{\beta}_{RR}$ are as follows respectively,

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$$\mathrm{MSE}(\hat{\beta}) = \sigma^2 \sum_{i=1}^{p} \frac{1}{\lambda_i} \tag{1.4}$$

$$\mathrm{MSE}\left(\hat{\beta}_{RR}\right) = Var\left(\hat{\beta}_{RR}\right) + \left[\mathrm{Bias}(\hat{\beta}_{RR})\right]^2$$
$$= \sigma^2 \sum_{i=1}^{p} \frac{\lambda_i}{(\lambda_i + k)^2} + k^2 \beta'(X'X + kI_p)^{-2}\beta \tag{1.5}$$

where $\lambda_i$ 's are eigenvalues of the matrix $X'X$ and $\sigma^2$ is the error variance.

Hoerl and Kennard, 1970b showed the properties of this function in detail. They concluded that the total variance decreases and the squared bias increases as $k$ increases. The variance function is monotonically decreasing and the squared bias function is monotonically increasing. Thus, there is the probability that some $k$ exists such that the MSE for $\hat{\beta}_{RR}$ is less than MSE for the usual $\hat{\beta}$ (Hoerl and Kennard, 1970ab).

We know that $k$ is estimated from the observed data. There are many papers proposing different ridge parameters in the literature. In recent papers, these parameters have been compared with the one proposed by Hoerl et al., 1975 and each other. After Hoerl and Kennard, 1970b, many researchers studied this area and proposed different estimates of the ridge parameter. Some of them are McDonald and Galarneau (1975), Lawless and Wang (1976), Saleh and Kibria (1993), Liu and Gao (2011), Kibria (2003), Khalaf and Shukur (2005), Alkhamisi et al. (2006), Adnan et al. (2006), Yan (2008), Yan and Zhao (2009), Muniz and Kibria (2009), Mansson et al. (2010), Al-Hassan (2010), Muniz et al. (2012), Asar et al. (2014) and Dorugade (2014).

The purpose of this article is to study much of the parameters in the literature and propose some new ones and also make a comparison between them by conducting a Monte Carlo experiment. The comparison criterion is based on the mean squared properties.

The article is organized as follows. In Section 2, we present the methodology of different estimators and give some new estimators. A Monte Carlo simulation has been provided in Section 3. Results of the simulation are discussed in Section 4. In Section 5, an application of the estimators is given. Finally, we give a summary and conclusion.

## 2. Model and estimators

Firstly we write the general model (1.1) in canonical form. Suppose that there exits an orthogonal matrix D we apply a transformation such that

$$D(X'X)D' = \Lambda = diag(\lambda_1, \lambda_2, \ldots, \lambda_p) \tag{2.1}$$

where D is a $p \times p$ orthogonal matrix and $\lambda_1 \geqslant \lambda_2 \geqslant \cdots \geqslant \lambda_p$. If we substitute $Z = XD$ and $\alpha = D'\beta$ in the model (1.1), then the model may be rewritten as

$$Y = Z\alpha + \varepsilon \tag{2.2}$$

where $Z'Z = \Lambda$.

Thus, the ridge estimator of $\alpha$ becomes $\hat{\alpha}_{RR} = (Z'Z + kI_p)^{-1} Z'Y$. It is stated in Hoerl and Kennard, 1970a that the value of $k$ minimizing the $\mathrm{MSE}(\hat{\alpha}_{RR})$ is

$$k_i = \frac{\hat{\sigma}^2}{\alpha_i^2}. \tag{2.3}$$

As seen in the formula (2.3), $k$ depends on the unknown parameters $\sigma^2$ and $\alpha$. Hence we use the estimators $\hat{\sigma}^2$ and $\hat{\alpha}$ due to Hoerl and Kennard, 1970b and get

$$k_i = \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}. \tag{2.4}$$

### 2.1. Proposed estimators

In this section, we review some of the ridge estimators suggested earlier and propose some new ones. The list of estimators with which we will compare ours is given below:

$$(1)\ k_1 = k_{HK} = \frac{\hat{\sigma}^2}{\hat{\alpha}_{\max}^2} \quad \text{(Hoerl \& Kennard, 1970a)} \tag{2.5}$$

where $\hat{\alpha}_{\max}$ is the maximum element of $\hat{\alpha}$.

$$(2)\ k_2 = \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p} \lambda_i \hat{\alpha}_i^2}, i = 1, 2, \ldots, p \quad \text{(Lawless \& Wang, 1976)} \tag{2.6}$$

which is proposed from the Bayesian point of view.

$$(3)\ k_3 = median\left(\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}\right), i = 1, 2, \ldots, p \quad \text{(Kibria, 2003)} \tag{2.7}$$

which is the median of $\hat{k}_i = \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}$.

$$(4)\ k_4 = \frac{2\hat{\sigma}^2}{\lambda_{\max}\left(\prod \hat{\alpha}_i^2\right)^{1/p}}, i = 1, 2, \ldots, p \quad \text{(Dorugade, 2014)} \tag{2.8}$$

which is the geometric mean of $\hat{k}_i = \frac{2\hat{\sigma}^2}{\lambda_{\max}\hat{\alpha}_i^2}$.

$$(5)\ k_5 = \frac{2p}{\lambda_{\max}} \frac{\hat{\sigma}^2}{\sum_{i=1}^{p} \hat{\alpha}_i^2}, i = 1, 2, \ldots, p \quad \text{(Dorugade, 2014)} \tag{2.9}$$

which is the harmonic mean of $\hat{k}_i = \frac{2\hat{\sigma}^2}{\lambda_{\max}\hat{\alpha}_i^2}$.

A sufficient condition that $\mathrm{MSE}(\hat{\alpha}_{RR}) < \mathrm{MSE}(\hat{\alpha})$ is given by Hoerl and Kennard (1970a,b) such that $k < k_{HK} = \frac{\hat{\sigma}^2}{\hat{\alpha}_{\max}^2}$. A quick survey shows us that some of the existing ridge parameters are smaller than $k_{HK}$. However, if we try the estimators larger than $k_{HK}$, we observe that one can also have better estimators in sense of MSE.

In the figure given by Hoerl and Kennard (1970a,b), it is obvious that the first derivative of the function $\mathrm{MSE}(\hat{\alpha}_{RR})$ is negative when the value of $k_{HK}$ is used as the biasing parameter. Therefore, any estimator satisfying $0 < k < k_{HK}$ gives us a negative derivative. However, if we examine the intersection point of the variance and the squared bias functions, we see that it is absolutely greater than $k_{HK}$. Thus, one can find estimators such that the first derivative of the $\mathrm{MSE}(\hat{\alpha}_{RR})$ function is positive and being greater than $k_{HK}$. There are greater estimators than $k_{HK}$ in the literature, for example see Alkhamisi and Shukur, 2007 for the estimators $k_{NAS}$ and $k_{AS}$.

It should also be pointed out that the optimal selection process of the parameter $k$ in ridge regression cannot be truly provided from the theoretical point of view. Actually, this is an open problem to researchers. Thus we suggest some estimators which are modifications of $k_K = \frac{1}{p}\sum_{i=1}^{p} \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}$ proposed in Kibria, 2003 $k_D = \frac{2p}{\lambda_{\max}} \frac{\hat{\sigma}^2}{\sum_{i=1}^{p} \hat{\alpha}_i^2}$ and $k_D = \frac{2p}{\lambda_{\max}} \frac{\hat{\sigma}^2}{\sum_{i=1}^{p} \hat{\alpha}_i^2}$ proposed in Dorugade, 2014. We apply some transformations and we fol-

**Table 1** The AMSE as a function of $\sigma^2$.

| $n$ | 20 | | | | 50 | | | | 100 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $p = 4, \rho = 0.75$ | | | | | | | | | | | | |
| $\sigma^2$ | 0.1 | 0.5 | 1.0 | 5.0 | 0.1 | 0.5 | 1.0 | 5.0 | 0.1 | 0.5 | 1.0 | 5.0 |
| $k_1$ | 0.8018 | 4.3489 | 7.5183 | 39.0185 | 0.7438 | 3.7920 | 6.8246 | 27.9983 | 0.7029 | 2.9245 | 6.1673 | 23.5762 |
| $k_2$ | 0.7727 | 4.7676 | 8.3778 | 44.2431 | 0.7133 | 4.0857 | 7.4317 | 28.0331 | 0.6664 | 3.0179 | 6.4671 | 23.1509 |
| $k_3$ | 0.7846 | 5.0495 | 9.1799 | 48.9552 | 0.7428 | 4.3901 | 8.1411 | 31.7462 | 0.6884 | 3.3637 | 7.1451 | 27.2164 |
| $k_4$ | 0.7167 | 4.4135 | 8.1842 | 44.2272 | 0.6709 | 3.9050 | 7.2784 | 28.5271 | 0.6226 | 3.0754 | 6.3872 | 25.1134 |
| $k_5$ | 0.7088 | 3.6649 | 6.0314 | 29.9480 | 0.6477 | 3.2296 | 5.5757 | 22.1414 | 0.6069 | 2.4728 | 5.0739 | 18.7134 |
| $k_{N1}$ | 0.6910 | 3.6451 | 6.0659 | 30.2548 | 0.6325 | 3.2132 | 5.5900 | 22.0714 | 0.5918 | 2.4698 | 5.0680 | 18.7983 |
| $k_{N2}$ | 0.7298 | 3.7024 | 6.0289 | 29.7302 | 0.6691 | 3.2641 | 5.5984 | 22.4143 | 0.6299 | 2.5016 | 5.1228 | 18.8006 |
| $k_{N3}$ | 0.7245 | 3.6846 | 6.0369 | 29.7471 | 0.6706 | 3.2559 | 5.6112 | 22.5370 | 0.6343 | 2.5223 | 5.1435 | 19.0303 |
| $k_{N4}$ | 0.6862 | 4.3086 | 7.3445 | 36.7735 | 0.6309 | 3.7810 | 6.7037 | 25.8954 | 0.5828 | 2.8875 | 5.9905 | 22.0626 |
| OLS | 1.4625 | 8.6222 | 14.2958 | 71.7297 | 1.2800 | 7.3098 | 12.6382 | 50.2923 | 1.1755 | 5.4638 | 11.4942 | 42.1464 |
| $p = 4, \rho = 0.85$ | | | | | | | | | | | | |
| $\sigma^2$ | 0.1 | 0.5 | 1.0 | 5.0 | 0.1 | 0.5 | 1.0 | 5.0 | 0.1 | 0.5 | 1.0 | 5.0 |
| $k_1$ | 2.5925 | 7.9881 | 12.6128 | 102.2997 | 1.4107 | 5.3567 | 11.8893 | 79.6890 | 1.2014 | 4.4455 | 10.2231 | 60.0734 |
| $k_2$ | 2.8936 | 10.1650 | 15.3204 | 137.0792 | 1.4823 | 6.1005 | 14.3277 | 103.9478 | 1.2273 | 4.8987 | 11.7653 | 74.9186 |
| $k_3$ | 2.9571 | 10.6945 | 15.7596 | 137.3515 | 1.5931 | 6.3183 | 14.4882 | 98.3121 | 1.2976 | 5.1854 | 12.1973 | 76.2568 |
| $k_4$ | 2.6016 | 9.4165 | 13.5265 | 121.9175 | 1.4319 | 5.4256 | 12.2743 | 84.8187 | 1.1766 | 4.5210 | 10.4937 | 66.7519 |
| $k_5$ | 2.2860 | 6.0829 | 9.6697 | 72.4183 | 1.2541 | 4.4396 | 9.4002 | 60.2166 | 1.0848 | 3.6822 | 7.9935 | 45.1846 |
| $k_{N1}$ | 2.2572 | 6.1846 | 9.7280 | 73.4788 | 1.2377 | 4.4184 | 9.3862 | 59.9767 | 1.0650 | 3.6675 | 7.9889 | 45.3840 |
| $k_{N2}$ | 2.2968 | 5.9923 | 9.6551 | 71.7758 | 1.2685 | 4.4704 | 9.4386 | 60.4528 | 1.1005 | 3.7128 | 8.0291 | 45.1771 |
| $k_{N3}$ | 2.2387 | 6.0057 | 9.6579 | 72.5659 | 1.2449 | 4.4367 | 9.3929 | 59.9748 | 1.0752 | 3.6966 | 8.0014 | 45.1750 |
| $k_{N4}$ | 2.6230 | 7.9527 | 12.1402 | 96.0898 | 1.4050 | 5.3006 | 11.4860 | 73.5664 | 1.1539 | 4.3835 | 9.7517 | 56.7623 |
| OLS | 5.3294 | 14.4634 | 23.1875 | 178.5352 | 2.7670 | 10.1480 | 21.6767 | 139.8967 | 2.2929 | 8.1819 | 18.1937 | 104.3610 |
| $p = 4, \rho = 0.95$ | | | | | | | | | | | | |
| $\sigma^2$ | 0.1 | 0.5 | 1.0 | 5.0 | 0.1 | 0.5 | 1.0 | 5.0 | 0.1 | 0.5 | 1.0 | 5.0 |
| $k_1$ | 4.4843 | 17.6719 | 43.2137 | 285.2298 | 3.6713 | 15.2385 | 39.8144 | 211.9142 | 3.0023 | 13.8284 | 27.6675 | 187.9357 |
| $k_2$ | 5.4332 | 24.3150 | 62.0304 | 428.1496 | 4.3521 | 20.3761 | 57.6365 | 317.5391 | 3.5398 | 18.2148 | 37.9291 | 276.4823 |
| $k_3$ | 5.2084 | 21.7618 | 53.5358 | 369.9291 | 4.2803 | 18.1883 | 51.3576 | 277.0999 | 3.6190 | 16.7532 | 34.8457 | 241.7900 |
| $k_4$ | 4.4122 | 17.5500 | 43.9138 | 326.1944 | 3.6350 | 14.6845 | 42.0855 | 242.7515 | 3.1331 | 13.6560 | 28.8147 | 212.4845 |
| $k_5$ | 3.8087 | 13.6617 | 32.3764 | 205.8630 | 3.1348 | 11.9416 | 29.4144 | 153.8375 | 2.5679 | 10.7092 | 20.8151 | 137.6582 |
| $k_{N1}$ | 3.7673 | 13.5906 | 32.2286 | 206.1835 | 3.1006 | 11.8395 | 29.4070 | 153.9551 | 2.5506 | 10.6571 | 20.7991 | 137.6976 |
| $k_{N2}$ | 3.8173 | 13.6882 | 32.4253 | 205.9081 | 3.1437 | 11.9751 | 29.4385 | 153.9099 | 2.5752 | 10.7345 | 20.8414 | 137.7477 |
| $k_{N3}$ | 3.7398 | 13.6022 | 32.3259 | 209.1952 | 3.0774 | 11.7948 | 29.6282 | 155.4830 | 2.5457 | 10.6581 | 20.8693 | 138.7788 |
| $k_{N4}$ | 4.4834 | 16.7420 | 39.8726 | 263.3648 | 3.6897 | 14.3511 | 37.4322 | 196.8083 | 3.0924 | 13.1481 | 26.1427 | 174.9628 |
| OLS | 9.0396 | 32.5136 | 78.5206 | 514.2492 | 7.1454 | 27.9423 | 68.4895 | 361.1289 | 5.7255 | 24.6632 | 48.0141 | 321.5286 |
| $p = 8, \rho = 0.75$ | | | | | | | | | | | | |
| $\sigma^2$ | 0.1 | 0.5 | 1.0 | 5.0 | 0.1 | 0.5 | 1.0 | 5.0 | 0.1 | 0.5 | 1.0 | 5.0 |
| $k_1$ | 2.2324 | 7.9806 | 20.7148 | 98.3940 | 1.7880 | 7.4095 | 15.8289 | 89.4041 | 1.7462 | 7.2615 | 13.0052 | 82.4673 |
| $k_2$ | 2.2673 | 8.1524 | 25.6294 | 122.2006 | 1.8878 | 7.8641 | 18.0645 | 108.6631 | 1.7441 | 7.6161 | 13.9494 | 97.8043 |
| $k_3$ | 2.6692 | 8.7870 | 27.0009 | 125.1359 | 2.2413 | 8.5847 | 19.2593 | 111.2952 | 1.9512 | 8.2726 | 15.0922 | 100.5385 |
| $k_4$ | 2.3364 | 7.6846 | 23.7623 | 107.8760 | 2.0024 | 7.5203 | 16.8049 | 96.3113 | 1.7414 | 7.3143 | 13.2117 | 86.9273 |
| $k_5$ | 2.1026 | 7.7269 | 17.9763 | 85.8827 | 1.6169 | 7.0094 | 14.4084 | 79.5819 | 1.6307 | 6.8876 | 12.0654 | 74.4692 |
| $k_{N1}$ | 2.0667 | 7.5343 | 17.9416 | 84.9353 | 1.6063 | 6.8763 | 14.2413 | 78.6360 | 1.6036 | 6.7574 | 11.8623 | 73.4395 |
| $k_{N2}$ | 2.0581 | 7.4499 | 17.9446 | 84.9656 | 1.6090 | 6.8610 | 14.2226 | 78.3751 | 1.5976 | 6.7238 | 11.8579 | 73.1182 |
| $k_{N3}$ | 2.1998 | 8.1117 | 18.5642 | 90.3647 | 1.6988 | 7.4657 | 15.0726 | 82.3965 | 1.7116 | 7.2623 | 12.8917 | 77.3164 |
| $k_{N4}$ | 2.3118 | 7.9974 | 22.2105 | 101.7014 | 1.8764 | 7.6855 | 16.6246 | 92.7855 | 1.7125 | 7.4613 | 13.3351 | 85.0792 |
| OLS | 4.2967 | 15.5416 | 38.8491 | 185.4425 | 3.1334 | 13.8163 | 28.8404 | 159.1808 | 2.9932 | 13.1563 | 23.3809 | 145.3175 |
| $p = 8, \rho = 0.85$ | | | | | | | | | | | | |
| $\sigma^2$ | 0.1 | 0.5 | 1.0 | 5.0 | 0.1 | 0.5 | 1.0 | 5.0 | 0.1 | 0.5 | 1.0 | 5.0 |
| $k_1$ | 3.1444 | 21.4056 | 54.3085 | 202.0053 | 2.5682 | 17.7387 | 36.0596 | 156.8762 | 2.4620 | 10.8207 | 25.1449 | 119.7675 |
| $k_2$ | 3.3571 | 28.3796 | 75.8952 | 284.6723 | 2.6303 | 22.1753 | 47.4561 | 211.9725 | 2.5101 | 12.2457 | 30.9778 | 154.2272 |
| $k_3$ | 3.8979 | 29.1936 | 72.4889 | 275.8329 | 2.9825 | 22.5715 | 46.6600 | 201.8615 | 2.7762 | 12.8455 | 31.2573 | 150.6953 |
| $k_4$ | 3.3834 | 25.8376 | 59.1447 | 238.7088 | 2.6130 | 19.3626 | 39.2020 | 172.1409 | 2.4645 | 11.0033 | 26.5278 | 128.3152 |
| $k_5$ | 2.9414 | 17.5096 | 47.5644 | 170.9078 | 2.4781 | 15.6459 | 31.5818 | 139.3364 | 2.3925 | 10.2494 | 22.8063 | 108.1431 |
| $k_{N1}$ | 2.8966 | 17.6051 | 46.9861 | 170.3857 | 2.4293 | 15.5167 | 31.2769 | 137.6626 | 2.3448 | 10.0590 | 22.5121 | 106.6644 |
| $k_{N2}$ | 2.8683 | 17.6876 | 46.4738 | 170.4134 | 2.3926 | 15.4309 | 31.0563 | 136.3521 | 2.3049 | 9.9242 | 22.3054 | 105.6248 |
| $k_{N3}$ | 2.9750 | 17.4701 | 46.9108 | 171.0634 | 2.5064 | 15.6132 | 31.4888 | 139.1297 | 2.4100 | 10.4023 | 22.9707 | 109.0800 |
| $k_{N4}$ | 3.3558 | 23.5494 | 57.9280 | 219.3679 | 2.6441 | 19.0878 | 38.4865 | 165.6201 | 2.4915 | 11.3337 | 26.5494 | 125.2557 |
| OLS | 6.0621 | 39.0492 | 101.3570 | 375.4153 | 4.6611 | 31.7765 | 63.3548 | 279.4298 | 4.3699 | 19.6167 | 44.6007 | 209.6424 |

(*continued on next page*)

**Table 1** (*continued*)

| $n$ | 20 | | | | 50 | | | | 100 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $p = 8, \rho = 0.95$ | | | | | | | | | | | | |
| $\sigma^2$ | 0.1 | 0.5 | 1.0 | 5.0 | 0.1 | 0.5 | 1.0 | 5.0 | 0.1 | 0.5 | 1.0 | 5.0 |
| $k_1$ | 10.1517 | 62.3589 | 256.3495 | 510.1337 | 8.3145 | 43.598 | 114.5677 | 443.13 | 7.0777 | 38.516 | 72.2927 | 441.6436 |
| $k_2$ | 12.4626 | 91.64 | 394.4763 | 797.0015 | 10.055 | 61.718 | 175.5814 | 679.2201 | 8.1842 | 53.2412 | 104.6281 | 677.2535 |
| $k_3$ | 12.9914 | 84.9505 | 367.1545 | 668.6289 | 10.5799 | 55.2194 | 150.651 | 586.6407 | 8.6537 | 50.1042 | 92.8478 | 598.2785 |
| $k_4$ | 10.7853 | 69.2146 | 320.9709 | 554.7026 | 9.163 | 44.3726 | 122.6003 | 497.4623 | 7.4245 | 41.7137 | 76.8874 | 516.3498 |
| $k_5$ | 9.3234 | 53.168 | 195.5032 | 464.0262 | 7.4015 | 40.5554 | 103.2034 | 388.1869 | 6.6582 | 33.5896 | 65.7234 | 376.9323 |
| $k_{N1}$ | 9.1688 | 52.6306 | 196.4644 | 455.3866 | 7.3301 | 39.7718 | 101.5887 | 383.3861 | 6.5494 | 33.2534 | 64.7785 | 374.3269 |
| $k_{N2}$ | 8.9755 | 52.1332 | 199.8805 | 443.7877 | 7.2614 | 38.6766 | 99.4564 | 378.0306 | 6.4089 | 32.921 | 63.6095 | 372.6178 |
| $k_{N3}$ | 8.9577 | 52.1043 | 201.9508 | 441.7225 | 7.2644 | 38.5061 | 98.9504 | 377.8803 | 6.4152 | 32.9412 | 63.6302 | 372.6184 |
| $k_{N4}$ | 10.9554 | 67.0785 | 283.3606 | 534.8243 | 9.0699 | 45.7474 | 121.9183 | 472.5697 | 7.5963 | 41.3113 | 76.988 | 481.2463 |

low Khalaf and Shukur (2005) and Alkhamisi and Shukur (2007) in order to get some estimators being greater than $k_{HK}$ and having better performances. The first two estimators are smaller than $k_{HK}$ and others are greater than it.

The following are our proposed estimators:

$$(1)\ k_{N1} = \frac{\sqrt{5}p}{\lambda_{\max}} \frac{\hat{\sigma}^2}{\sum_{i=1}^{p} \hat{\alpha}_i^2} \quad (2.10)$$

We suggest the modification by multiplying $\frac{\lambda_{\max}}{\sqrt{5}}$ to the denominator of (2.4). Thus the suggested estimator is $\frac{\sqrt{5}\hat{\sigma}^2}{\lambda_{\max}\hat{\alpha}_i^2}$. This is an estimator having a denominator greater than that of Hoerl and Kennard, 1970a. Thus, we can write $\frac{\hat{\sigma}^2}{\hat{\alpha}^2} \geqslant \frac{\sqrt{5}\hat{\sigma}^2}{\lambda_{\max}\hat{\alpha}_i^2}, i = 1, 2, \ldots, p$. Finally, we use harmonic mean function and get the new estimator given in Eq. (2.10).

$$(2)\ k_{N2} = \frac{p}{\sqrt{\lambda_{\max}}} \frac{\hat{\sigma}^2}{\sum_{i=1}^{p} \hat{\alpha}_i^2} \quad (2.11)$$

Similar to the above discussion, we multiply the denominator of (2.4) by $\lambda_{\max}$ and we get $\frac{\hat{\sigma}^2}{\lambda_{\max}\hat{\alpha}_i^2}$. Again, we have $\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2} \geqslant \frac{\hat{\sigma}^2}{\lambda_{\max}\hat{\alpha}_i^2}$ showing that this new estimator is clearly smaller than (2.4). Taking the harmonic mean, we finally get the new estimator given in (2.11).

$$(3)\ k_{N3} = \frac{2p}{\sum_{i=1}^{p} \left(\lambda_i^{1/4}\right)} \frac{\hat{\sigma}^2}{\sum_{i=1}^{p} \hat{\alpha}i^2} \quad (2.12)$$

$$(4)\ k_{N4} = \frac{2p}{\sqrt{\sum_{i=1}^{p} \lambda_i}} \frac{\hat{\sigma}^2}{\sum_{i=1}^{p} \hat{\alpha}_i^2} \quad (2.13)$$

We have $\sum_{i=}^{p} \lambda_i > p$ because the matrix $X'X$ is in the correlation form. Thus, the new proposed estimators $k_{N3}$ and $k_{N4}$ are definitely greater than $k_{HK}$. All the above parameters will be compared by a Monte Carlo simulation and the whole process is explained in Section 3.

## 3. The Monte Carlo simulation

In this section, a Monte Carlo simulation has been conducted to compare the performances of the estimators. There are two criteria used to design a good Monte Carlo simulation. One of them is to specify what factors are expected to affect the properties of the estimators and the other is to determine the criterion of judgment. We decided that the effective factors are the data size $n$, the number of explanatory variables $p$, the correlation between the explanatory variables $\rho$ and the variance of error terms $\sigma^2$. Mean squared error (MSE) will be the criterion to compare the performances of the estimators. In the simulation, we examined the average MSE (AMSE) of the ridge parameters. Now, we give details of the study.

The mean squared error of the ridge estimator $\hat{\beta}_R$ is

$$\mathrm{MSE}\left(\hat{\beta}_R\right) = Var\left(\hat{\beta}_R\right) + \left[\mathrm{Bias}(\hat{\beta}_R)\right]^2 = \hat{\sigma}^2 \sum_{i=1}^{p} \frac{\lambda_i}{(\lambda_i + k)^2} + k^2 \sum_{i=1}^{p} \frac{\hat{\alpha}_i^2}{(\lambda_i + k)^2} \quad (3.1)$$

Although we reviewed 41 different estimators for estimating the ridge parameter $k$, we finally consider $k_1, k_2, k_3, k_4, k_5$ from the literature and new proposed $k_{N1}, k_{N2}, k_{N3}$ and $k_{N4}$ of them.

The true model $Y = X\beta + \varepsilon$ is considered with independent $\varepsilon \sim N(0, \sigma^2)$ and $\beta$ is chosen such that $\beta'\beta = 1$ since Newhouse and Oman, 1971 stated that if $\beta$ is taken to be the eigenvector of the largest eigenvalue of the matrix $X'X$ then the MSE is minimized.

To generate the explanatory variables, we used the following commonly used process:

$x_{ji} = (1 - \rho^2)^{1/2} z_{ji} + \rho z_{jp}, j = 1, 2, \ldots, n$ and $i = 1, 2, \ldots, p$ where $\rho^2$ represents the correlation between the explanatory variables and $z_{ij}$'s are independent, random numbers following the standard normal distribution. Also, the dependent variable $Y$ is generated by

$Y_j = \beta_1 x_{j1} + \beta_2 x_{j2} + \ldots + \beta_p x_{jp} + \varepsilon_j, \quad j = 1, 2, \ldots, n$ where $\varepsilon_j$'s are independent normal pseudorandom numbers with zero mean and variance $\sigma^2$.

Here, we consider the cases $n = 20, 50, 100$; $\rho = 0.75, 0.85, 0.95$; $p = 4, 8$ and $\sigma^2 = 0.1, 0.5, 1.0, 5.0$. After generating the explanatory variables $X$ and the dependent variable $Y$, we standardized both of them so that $X'X$ and $X'Y$ are in the correlation form.

For the values of $n$, $p$, $\rho$ and $\sigma^2$, the experiment was repeated 10.000 times by generating the error terms in the Eq. (1.1). After this procedure, for each replicate $\mathrm{MSE}_{OLS}$, $\mathrm{MSE}_{RR}$ and the average mean squared error (AMSE) for each estimator are calculated for each of the values $(n, p, \rho, \sigma^2)$ such that

$$\mathrm{AMSE}(\hat{\alpha}) = \frac{1}{10000} \sum_{r=1}^{10000} \mathrm{MSE}(\hat{\alpha}) \quad (3.2)$$

**Table 2** The AMSE as a function of ρ.

| $\sigma^2$ | 0.1 | | | 0.5 | | | 1.0 | | | 5.0 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $n = 20, p = 4$ | | | | | | | | | | | | |
| $\rho$ | 0.75 | 0.85 | 0.95 | 0.75 | 0.75 | 0.85 | 0.95 | 0.75 | 0.75 | 0.85 | 0.95 | 0.75 |
| $k_1$ | 0.8018 | 2.5925 | 4.4843 | 4.3489 | 7.9881 | 17.6719 | 7.5183 | 12.6128 | 43.2137 | 39.0185 | 102.2997 | 285.2298 |
| $k_2$ | 0.7727 | 2.8936 | 5.4332 | 4.7676 | 10.1650 | 24.3150 | 8.3778 | 15.3204 | 62.0304 | 44.2431 | 137.0792 | 428.1496 |
| $k_3$ | 0.7846 | 2.9571 | 5.2084 | 5.0495 | 10.6945 | 21.7618 | 9.1799 | 15.7596 | 53.5358 | 48.9552 | 137.3515 | 369.9291 |
| $k_4$ | 0.7167 | 2.6016 | 4.4122 | 4.4135 | 9.4165 | 17.5500 | 8.1842 | 13.5265 | 43.9138 | 44.2272 | 121.9175 | 326.1944 |
| $k_5$ | 0.7088 | 2.2860 | 3.8087 | 3.6649 | 6.0829 | 13.6617 | 6.0314 | 9.6697 | 32.3764 | 29.9480 | 72.4183 | 205.8630 |
| $k_{N1}$ | 0.6910 | 2.2572 | 3.7673 | 3.6451 | 6.1846 | 13.5906 | 6.0659 | 9.7280 | 32.2286 | 30.2548 | 73.4788 | 206.1835 |
| $k_{N2}$ | 0.7298 | 2.2968 | 3.8173 | 3.7024 | 5.9923 | 13.6882 | 6.0289 | 9.6551 | 32.4253 | 29.7302 | 71.7758 | 205.9081 |
| $k_{N3}$ | 0.7245 | 2.2387 | 3.7398 | 3.6846 | 6.0057 | 13.6022 | 6.0369 | 9.6579 | 32.3259 | 29.7471 | 72.5659 | 209.1952 |
| $k_{N4}$ | 0.6862 | 2.6230 | 4.4834 | 4.3086 | 7.9527 | 16.7420 | 7.3445 | 12.1402 | 39.8726 | 36.7735 | 96.0898 | 263.3648 |
| OLS | 1.4625 | 5.3294 | 9.0396 | 8.6222 | 14.4634 | 32.5136 | 14.2958 | 23.1875 | 78.5206 | 71.7297 | 178.5352 | 514.2492 |
| $n = 50, p = 4$ | | | | | | | | | | | | |
| ρ | 0.75 | 0.85 | 0.95 | 0.75 | 0.75 | 0.85 | 0.95 | 0.75 | 0.75 | 0.85 | 0.95 | 0.75 |
| $k_1$ | 0.7438 | 1.4107 | 3.6713 | 3.7920 | 5.3567 | 15.2385 | 6.8246 | 11.8893 | 39.8144 | 27.9983 | 79.6890 | 211.9142 |
| $k_2$ | 0.7133 | 1.4823 | 4.3521 | 4.0857 | 6.1005 | 20.3761 | 7.4317 | 14.3277 | 57.6365 | 28.0331 | 103.9478 | 317.5391 |
| $k_3$ | 0.7428 | 1.5931 | 4.2803 | 4.3901 | 6.3183 | 18.1883 | 8.1411 | 14.4882 | 51.3576 | 31.7462 | 98.3121 | 277.0999 |
| $k_4$ | 0.6709 | 1.4319 | 3.6350 | 3.9050 | 5.4256 | 14.6845 | 7.2784 | 12.2743 | 42.0855 | 28.5271 | 84.8187 | 242.7515 |
| $k_5$ | 0.6477 | 1.2541 | 3.1348 | 3.2296 | 4.4396 | 11.9416 | 5.5757 | 9.4002 | 29.4144 | 22.1414 | 60.2166 | 153.8375 |
| $k_{N1}$ | 0.6325 | 1.2377 | 3.1006 | 3.2132 | 4.4184 | 11.8395 | 5.5900 | 9.3862 | 29.4070 | 22.0714 | 59.9767 | 153.9551 |
| $k_{N2}$ | 0.6691 | 1.2685 | 3.1437 | 3.2641 | 4.4704 | 11.9751 | 5.5984 | 9.4386 | 29.4385 | 22.4143 | 60.4528 | 153.9099 |
| $k_{N3}$ | 0.6706 | 1.2449 | 3.0774 | 3.2559 | 4.4367 | 11.7948 | 5.6112 | 9.3929 | 29.6282 | 22.5370 | 59.9748 | 155.4830 |
| $k_{N4}$ | 0.6309 | 1.4050 | 3.6897 | 3.7810 | 5.3006 | 14.3511 | 6.7037 | 11.4860 | 37.4322 | 25.8954 | 73.5664 | 196.8083 |
| OLS | 1.2800 | 2.7670 | 7.1454 | 7.3098 | 10.1480 | 27.9423 | 12.6382 | 21.6767 | 68.4895 | 50.2923 | 139.8967 | 361.1289 |
| $n = 100, p = 4$ | | | | | | | | | | | | |
| $\rho$ | 0.75 | 0.85 | 0.95 | 0.75 | 0.75 | 0.85 | 0.95 | 0.75 | 0.75 | 0.85 | 0.95 | 0.75 |
| $k_1$ | 0.7029 | 1.2014 | 3.0023 | 2.9245 | 4.4455 | 13.8284 | 6.1673 | 10.2231 | 27.6675 | 23.5762 | 60.0734 | 187.9357 |
| $k_2$ | 0.6664 | 1.2273 | 3.5398 | 3.0179 | 4.8987 | 18.2148 | 6.4671 | 11.7653 | 37.9291 | 23.1509 | 74.9186 | 276.4823 |
| $k_3$ | 0.6884 | 1.2976 | 3.6190 | 3.3637 | 5.1854 | 16.7532 | 7.1451 | 12.1973 | 34.8457 | 27.2164 | 76.2568 | 241.7900 |
| $k_4$ | 0.6226 | 1.1766 | 3.1331 | 3.0754 | 4.5210 | 13.6560 | 6.3872 | 10.4937 | 28.8147 | 25.1134 | 66.7519 | 212.4845 |
| $k_5$ | 0.6069 | 1.0848 | 2.5679 | 2.4728 | 3.6822 | 10.7092 | 5.0739 | 7.9935 | 20.8151 | 18.7134 | 45.1846 | 137.6582 |
| $k_{N1}$ | 0.5918 | 1.0650 | 2.5506 | 2.4698 | 3.6675 | 10.6571 | 5.0680 | 7.9889 | 20.7991 | 18.7983 | 45.3840 | 137.6976 |
| $k_{N2}$ | 0.6299 | 1.1005 | 2.5752 | 2.5016 | 3.7128 | 10.7345 | 5.1228 | 8.0291 | 20.8414 | 18.8006 | 45.1771 | 137.7477 |
| $k_{N3}$ | 0.6343 | 1.0752 | 2.5457 | 2.5223 | 3.6966 | 10.6581 | 5.1435 | 8.0014 | 20.8693 | 19.0303 | 45.1750 | 138.7788 |
| $k_{N4}$ | 0.5828 | 1.1539 | 3.0924 | 2.8875 | 4.3835 | 13.1481 | 5.9905 | 9.7517 | 26.1427 | 22.0626 | 56.7623 | 174.9628 |
| OLS | 1.1755 | 2.2929 | 5.7255 | 5.4638 | 8.1819 | 24.6632 | 11.4942 | 18.1937 | 48.0141 | 42.1464 | 104.3610 | 321.5286 |
| $n = 20, p = 8$ | | | | | | | | | | | | |
| $\rho$ | 0.75 | 0.85 | 0.95 | 0.75 | 0.85 | 0.95 | 0.75 | 0.85 | 0.95 | 0.75 | 0.85 | 0.95 |
| $k_1$ | 2.2324 | 3.1444 | 10.1517 | 7.9806 | 21.4056 | 62.3589 | 20.7148 | 54.3085 | 256.3495 | 98.3940 | 202.0053 | 510.1337 |
| $k_2$ | 2.2673 | 3.3571 | 12.4626 | 8.1524 | 28.3796 | 91.6400 | 25.6294 | 75.8952 | 394.4763 | 122.2006 | 284.6723 | 797.0015 |
| $k_3$ | 2.6692 | 3.8979 | 12.9914 | 8.7870 | 29.1936 | 84.9505 | 27.0009 | 72.4889 | 367.1545 | 125.1359 | 275.8329 | 668.6289 |
| $k_4$ | 2.3364 | 3.3834 | 10.7853 | 7.6846 | 25.8376 | 69.2146 | 23.7623 | 59.1447 | 320.9709 | 107.8760 | 238.7088 | 554.7026 |
| $k_5$ | 2.1026 | 2.9414 | 9.3234 | 7.7269 | 17.5096 | 53.1680 | 17.9763 | 47.5644 | 195.5032 | 85.8827 | 170.9078 | 464.0262 |
| $k_{N1}$ | 2.0667 | 2.8966 | 9.1688 | 7.5343 | 17.6051 | 52.6306 | 17.9416 | 46.9861 | 196.4644 | 84.9353 | 170.3857 | 455.3866 |
| $k_{N2}$ | 2.0581 | 2.8683 | 8.9755 | 7.4499 | 17.6876 | 52.1332 | 17.9446 | 46.4738 | 199.8805 | 84.9656 | 170.4134 | 443.7877 |
| $k_{N3}$ | 2.1998 | 2.9750 | 8.9577 | 8.1117 | 17.4701 | 52.1043 | 18.5642 | 46.9108 | 201.9508 | 90.3647 | 171.0634 | 441.7225 |
| $k_{N4}$ | 2.3118 | 3.3558 | 10.9554 | 7.9974 | 23.5494 | 67.0785 | 22.2105 | 57.9280 | 283.3606 | 101.7014 | 219.3679 | 534.8243 |
| OLS | 4.2967 | 6.0621 | 19.5364 | 15.5416 | 39.0492 | 116.0329 | 38.8491 | 101.3570 | 453.3480 | 185.4425 | 375.4153 | 961.3745 |
| $n = 50, p = 8$ | | | | | | | | | | | | |
| $\rho$ | 0.75 | 0.85 | 0.95 | 0.75 | 0.75 | 0.85 | 0.95 | 0.75 | 0.75 | 0.85 | 0.95 | 0.75 |
| $k_1$ | 1.7880 | 2.5682 | 8.3145 | 7.4095 | 17.7387 | 43.5980 | 15.8289 | 36.0596 | 114.5677 | 89.4041 | 156.8762 | 443.1300 |
| $k_2$ | 1.8878 | 2.6303 | 10.0550 | 7.8641 | 22.1753 | 61.7180 | 18.0645 | 47.4561 | 175.5814 | 108.6631 | 211.9725 | 679.2201 |
| $k_3$ | 2.2413 | 2.9825 | 10.5799 | 8.5847 | 22.5715 | 55.2194 | 19.2593 | 46.6600 | 150.6510 | 111.2952 | 201.8615 | 586.6407 |
| $k_4$ | 2.0024 | 2.6130 | 9.1630 | 7.5203 | 19.3626 | 44.3726 | 16.8049 | 39.2020 | 122.6003 | 96.3113 | 172.1409 | 497.4623 |
| $k_5$ | 1.6169 | 2.4781 | 7.4015 | 7.0094 | 15.6459 | 40.5554 | 14.4084 | 31.5818 | 103.2034 | 79.5819 | 139.3364 | 388.1869 |
| $k_{N1}$ | 1.6063 | 2.4293 | 7.3301 | 6.8763 | 15.5167 | 39.7718 | 14.2413 | 31.2769 | 101.5887 | 78.6360 | 137.6626 | 383.3861 |
| $k_{N2}$ | 1.6090 | 2.3926 | 7.2614 | 6.8610 | 15.4309 | 38.6766 | 14.2226 | 31.0563 | 99.4564 | 78.3751 | 136.3521 | 378.0306 |
| $k_{N3}$ | 1.6988 | 2.5064 | 7.2644 | 7.4657 | 15.6132 | 38.5061 | 15.0726 | 31.4888 | 98.9504 | 82.3965 | 139.1297 | 377.8803 |
| $k_{N4}$ | 1.8764 | 2.6441 | 9.0699 | 7.6855 | 19.0878 | 45.7474 | 16.6246 | 38.4865 | 121.9183 | 92.7855 | 165.6201 | 472.5697 |
| OLS | 3.1334 | 4.6611 | 14.9055 | 13.8163 | 31.7765 | 78.4528 | 28.8404 | 63.3548 | 201.7302 | 159.1808 | 279.4298 | 780.6791 |

(*continued on next page*)

Table 2 (*continued*)

| $\sigma^2$ | 0.1 | | | 0.5 | | | 1.0 | | | 5.0 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $n = 100, p = 8$ | | | | | | | | | | | | |
| $\rho$ | 0.75 | 0.85 | 0.95 | 0.75 | 0.75 | 0.85 | 0.95 | 0.75 | 0.75 | 0.85 | 0.95 | 0.75 |
| $k_1$ | 1.7462 | 2.4620 | 7.0777 | 7.2615 | 10.8207 | 38.5160 | 13.0052 | 25.1449 | 72.2927 | 82.4673 | 119.7675 | 441.6436 |
| $k_2$ | 1.7441 | 2.5101 | 8.1842 | 7.6161 | 12.2457 | 53.2412 | 13.9494 | 30.9778 | 104.6281 | 97.8043 | 154.2272 | 677.2535 |
| $k_3$ | 1.9512 | 2.7762 | 8.6537 | 8.2726 | 12.8455 | 50.1042 | 15.0922 | 31.2573 | 92.8478 | 100.5385 | 150.6953 | 598.2785 |
| $k_4$ | 1.7414 | 2.4645 | 7.4245 | 7.3143 | 11.0033 | 41.7137 | 13.2117 | 26.5278 | 76.8874 | 86.9273 | 128.3152 | 516.3498 |
| $k_5$ | 1.6307 | 2.3925 | 6.6582 | 6.8876 | 10.2494 | 33.5896 | 12.0654 | 22.8063 | 65.7234 | 74.4692 | 108.1431 | 376.9323 |
| $k_{N1}$ | 1.6036 | 2.3448 | 6.5494 | 6.7574 | 10.0590 | 33.2534 | 11.8623 | 22.5121 | 64.7785 | 73.4395 | 106.6644 | 374.3269 |
| $k_{N2}$ | 1.5976 | 2.3049 | 6.4089 | 6.7238 | 9.9242 | 32.9210 | 11.8579 | 22.3054 | 63.6095 | 73.1182 | 105.6248 | 372.6178 |
| $k_{N3}$ | 1.7116 | 2.4100 | 6.4152 | 7.2623 | 10.4023 | 32.9412 | 12.8917 | 22.9707 | 63.6302 | 77.3164 | 109.0800 | 372.6184 |
| $k_{N4}$ | 1.7125 | 2.4915 | 7.5963 | 7.4613 | 11.3337 | 41.3113 | 13.3351 | 26.5494 | 76.9880 | 85.0792 | 125.2557 | 481.2463 |
| OLS | 2.9932 | 4.3699 | 12.6329 | 13.1563 | 19.6167 | 66.3018 | 23.3809 | 44.6007 | 127.7137 | 145.3175 | 209.6424 | 758.4431 |

and results are given in Tables 1–3. We also computed biases of the ridge parameters and reported results in Figs. 1–12.

## 4. Results of the simulation

According to the results of the simulation, we get the following Tables 1–3 which show the average mean squared error (AMSE) values for different numbers of observation, number of explanatory variables, variances and the correlation values. We also give some of our important findings in terms of figures especially for some of the cases in which $n$ or $\rho$ changes when the others are fixed. Additionally, we give the comparison of biases in terms of figures for similar consideration. We did not give the tables of biases since they are too large.

### *4.1. Comparison of the estimators according to the AMSEs*

#### *4.1.1. Comparison according to the variances $\sigma^2$*

In Table 1, we have given the average mean squared error values of the estimators as a function of the variances. We can see the change of AMSEs according to the variances of the errors ($\sigma^2$). It is obvious that when $\sigma^2$ increases, the AMSE of the estimators increases. For all of the cases, AMSE of the OLS estimator is larger than the AMSE of the new proposed ridge estimators. In most of the cases, the estimators $k_{N1}$, $k_{N2}$, $k_{N3}$, $k_{N4}$ dominate the estimators $k_1$, $k_2$ and $k_3$. However, the performance of the proposed estimators $k_{N2}$, $k_{N3}$ and $k_{N4}$ (at least one of them) are the best in all cases.

For given values $n = 20$, $p = 4$, $\rho = 0.75$ and $n = 20$, $p = 8$, $\rho = 0.75$, the performances of the estimators are given in Figs. 1 and 2 respectively. We can see from these figures that as $\sigma^2$ changes from 0.1 to 5.0, the AMSE values of the estimators increase. The number of explanatory variables $p$ has a great effect on multicollinearity. If there are more variables correlated in the model, the effect of collinearity increases. In these figures, there is a change in the number of explanatory variables. In the case of $p = 8$, the AMSEs are larger than the former case. Actually, changing $p = 4$ to $p = 8$, fixing $n$,$\sigma^2$ and ρ , we see that there is an increase in the AMSEs in all cases.

#### *4.1.2. Comparison according to the correlation $\rho$*

In Table 2, we have given the AMSE values as a function of the correlation $\rho$. If we fix $n$ and $p$, we generally see that the AMSE values increase when the correlation increases. The performances of the estimators $k_{N1}$, $k_{N2}$, $k_{N3}$ and $k_{N4}$ are better than the other estimators. For given values $n = 20$, $p = 4$, $\sigma^2 = 0.1$ and $n = 20$, $p = 4$, $\sigma^2 = 5.0$, performances of the estimators are given in Figs. 3 and 4 respectively.

According to these figures, for smaller values of $\sigma^2$, the change in the correlation gives a small increase in the AMSE values. For each combination of the sample size $n$ and the number of variables $p$, the smaller the correlation, the smaller the AMSE values. However, the change in the correlation gives a large increase in AMSE values when jumping from $\sigma^2 = 0.1$ to $\sigma^2 = 5.0$. In all situations the OLS estimator has a larger AMSE compared to all the ridge estimators.

#### *4.1.3. Comparison according to the sample size n*

In Table 3, we have given the AMSE values of the estimators as a function of the sample size $n$. If $p$ and $\rho$ are fixed, we generally see that the AMSE values decrease when the data size $n$ increases. The performances of the estimators $k_{N1}$, $k_{N2}$, $k_{N3}$ are again better than the rest of the estimators. Sometimes $k_5$ dominates one of $k_{N1}$, $k_{N2}$, $k_{N3}$ but not all of them. For given values $p = 8$ and $\rho = 0.85$ the performances of the estimators for $\sigma^2 = 0.1$ and $\sigma^2 = 1.0$ are given in Figs. 5 and 6 respectively. We can say that there is a big amount of increase in the AMSE when jumping from $\sigma^2 = 0.1$ to $\sigma^2 = 1.0$. We did not include the line of AMSE values of the OLS estimator in the graph because if it is included, the scale becomes very large so that the difference between the estimators could not be seen from the figures.

It is obvious from Figs. 5 and 6 that $k_{N3}$ is the best estimator for the given case and the AMSE decreases when the sample size increases. In general, when $p = 4$, $k_{N1}$ has the best performance and if $p = 8$, then $k_{N3}$ has the best performance for all cases.

### *4.2. Comparison of the estimators according to the biases*

In this simulation study, we also considered biases of estimators. We know that some of the researchers need small biased estimators while the others only need estimators having small MSE. In this section, we compare biases of some selected estimators having least biases in the simulation. We only provide some graphs and make our comments using them.

**Table 3** The AMSE as a function of n.

| $\sigma^2$ | 0.1 | | | 0.5 | | | 1.0 | | | 5.0 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho = 0.75, p = 4$ | | | | | | | | | | | | |
| $n$ | 20 | 50 | 100 | 20 | 50 | 100 | 20 | 50 | 100 | 20 | 50 | 100 |
| $k_1$ | 0.8018 | 0.7438 | 0.7029 | 4.3489 | 3.7920 | 2.9245 | 7.5183 | 6.8246 | 6.1673 | 39.0185 | 27.9983 | 23.5762 |
| $k_2$ | 0.7727 | 0.7133 | 0.6664 | 4.7676 | 4.0857 | 3.0179 | 8.3778 | 7.4317 | 6.4671 | 44.2431 | 28.0331 | 23.1509 |
| $k_3$ | 0.7846 | 0.7428 | 0.6884 | 5.0495 | 4.3901 | 3.3637 | 9.1799 | 8.1411 | 7.1451 | 48.9552 | 31.7462 | 27.2164 |
| $k_4$ | 0.7167 | 0.6709 | 0.6226 | 4.4135 | 3.9050 | 3.0754 | 8.1842 | 7.2784 | 6.3872 | 44.2272 | 28.5271 | 25.1134 |
| $k_5$ | 0.7088 | 0.6477 | 0.6069 | 3.6649 | 3.2296 | 2.4728 | 6.0314 | 5.5757 | 5.0739 | 29.9480 | 22.1414 | 18.7134 |
| $k_{N1}$ | 0.6910 | 0.6325 | 0.5918 | 3.6451 | 3.2132 | 2.4698 | 6.0659 | 5.5900 | 5.0680 | 30.2548 | 22.0714 | 18.7983 |
| $k_{N2}$ | 0.7298 | 0.6691 | 0.6299 | 3.7024 | 3.2641 | 2.5016 | 6.0289 | 5.5984 | 5.1228 | 29.7302 | 22.4143 | 18.8006 |
| $k_{N3}$ | 0.7245 | 0.6706 | 0.6343 | 3.6846 | 3.2559 | 2.5223 | 6.0369 | 5.6112 | 5.1435 | 29.7471 | 22.5370 | 19.0303 |
| $k_{N4}$ | 0.6862 | 0.6309 | 0.5828 | 4.3086 | 3.7810 | 2.8875 | 7.3445 | 6.7037 | 5.9905 | 36.7735 | 25.8954 | 22.0626 |
| OLS | 1.4625 | 1.2800 | 1.1755 | 8.6222 | 7.3098 | 5.4638 | 14.2958 | 12.6382 | 11.4942 | 71.7297 | 50.2923 | 42.1464 |
| $\rho = 0.85, p = 4$ | | | | | | | | | | | | |
| $n$ | 20 | 50 | 100 | 20 | 50 | 100 | 20 | 50 | 100 | 20 | 50 | 100 |
| $k_1$ | 2.5925 | 1.4107 | 1.2014 | 7.9881 | 5.3567 | 4.4455 | 12.6128 | 11.8893 | 10.2231 | 102.2997 | 79.6890 | 60.0734 |
| $k_2$ | 2.8936 | 1.4823 | 1.2273 | 10.1650 | 6.1005 | 4.8987 | 15.3204 | 14.3277 | 11.7653 | 137.0792 | 103.9478 | 74.9186 |
| $k_3$ | 2.9571 | 1.5931 | 1.2976 | 10.6945 | 6.3183 | 5.1854 | 15.7596 | 14.4882 | 12.1973 | 137.3515 | 98.3121 | 76.2568 |
| $k_4$ | 2.6016 | 1.4319 | 1.1766 | 9.4165 | 5.4256 | 4.5210 | 13.5265 | 12.2743 | 10.4937 | 121.9175 | 84.8187 | 66.7519 |
| $k_5$ | 2.2860 | 1.2541 | 1.0848 | 6.0829 | 4.4396 | 3.6822 | 9.6697 | 9.4002 | 7.9935 | 72.4183 | 60.2166 | 45.1846 |
| $k_{N1}$ | 2.2572 | 1.2377 | 1.0650 | 6.1846 | 4.4184 | 3.6675 | 9.7280 | 9.3862 | 7.9889 | 73.4788 | 59.9767 | 45.3840 |
| $k_{N2}$ | 2.2968 | 1.2685 | 1.1005 | 5.9923 | 4.4704 | 3.7128 | 9.6551 | 9.4386 | 8.0291 | 71.7758 | 60.4528 | 45.1771 |
| $k_{N3}$ | 2.2387 | 1.2449 | 1.0752 | 6.0057 | 4.4367 | 3.6966 | 9.6579 | 9.3929 | 8.0014 | 72.5659 | 59.9748 | 45.1750 |
| $k_{N4}$ | 2.6230 | 1.4050 | 1.1539 | 7.9527 | 5.3006 | 4.3835 | 12.1402 | 11.4860 | 9.7517 | 96.0898 | 73.5664 | 56.7623 |
| OLS | 5.3294 | 2.7670 | 2.2929 | 14.4634 | 10.1480 | 8.1819 | 23.1875 | 21.6767 | 18.1937 | 178.5352 | 139.8967 | 104.3610 |
| $\rho = 0.95, p = 4$ | | | | | | | | | | | | |
| $n$ | 20 | 50 | 100 | 20 | 50 | 100 | 20 | 50 | 100 | 20 | 50 | 100 |
| $k_1$ | 4.4843 | 3.6713 | 3.0023 | 17.6719 | 15.2385 | 13.8284 | 43.2137 | 39.8144 | 27.6675 | 285.2298 | 211.9142 | 187.9357 |
| $k_2$ | 5.4332 | 4.3521 | 3.5398 | 24.3150 | 20.3761 | 18.2148 | 62.0304 | 57.6365 | 37.9291 | 428.1496 | 317.5391 | 276.4823 |
| $k_3$ | 5.2084 | 4.2803 | 3.6190 | 21.7618 | 18.1883 | 16.7532 | 53.5358 | 51.3576 | 34.8457 | 369.9291 | 277.0999 | 241.7900 |
| $k_4$ | 4.4122 | 3.6350 | 3.1331 | 17.5500 | 14.6845 | 13.6560 | 43.9138 | 42.0855 | 28.8147 | 326.1944 | 242.7515 | 212.4845 |
| $k_5$ | 3.8087 | 3.1348 | 2.5679 | 13.6617 | 11.9416 | 10.7092 | 32.3764 | 29.4144 | 20.8151 | 205.8630 | 153.8375 | 137.6582 |
| $k_{N1}$ | 3.7673 | 3.1006 | 2.5506 | 13.5906 | 11.8395 | 10.6571 | 32.2286 | 29.4070 | 20.7991 | 206.1835 | 153.9551 | 137.6976 |
| $k_{N2}$ | 3.8173 | 3.1437 | 2.5752 | 13.6882 | 11.9751 | 10.7345 | 32.4253 | 29.4385 | 20.8414 | 205.9081 | 153.9099 | 137.7477 |
| $k_{N3}$ | 3.7398 | 3.0774 | 2.5457 | 13.6022 | 11.7948 | 10.6581 | 32.3259 | 29.6282 | 20.8693 | 209.1952 | 155.4830 | 138.7788 |
| $k_{N4}$ | 4.4834 | 3.6897 | 3.0924 | 16.7420 | 14.3511 | 13.1481 | 39.8726 | 37.4322 | 26.1427 | 263.3648 | 196.8083 | 174.9628 |
| OLS | 9.0396 | 7.1454 | 5.7255 | 32.5136 | 27.9423 | 24.6632 | 78.5206 | 68.4895 | 48.0141 | 514.2492 | 361.1289 | 321.5286 |
| $\rho = 0.75, p = 8$ | | | | | | | | | | | | |
| $n$ | 20 | 50 | 100 | 20 | 50 | 100 | 20 | 50 | 100 | 20 | 50 | 100 |
| $k_1$ | 2.2324 | 1.7880 | 1.7462 | 7.9806 | 7.4095 | 7.2615 | 20.7148 | 15.8289 | 13.0052 | 98.3940 | 89.4041 | 82.4673 |
| $k_2$ | 2.2673 | 1.8878 | 1.7441 | 8.1524 | 7.8641 | 7.6161 | 25.6294 | 18.0645 | 13.9494 | 122.2006 | 108.6631 | 97.8043 |
| $k_3$ | 2.6692 | 2.2413 | 1.9512 | 8.7870 | 8.5847 | 8.2726 | 27.0009 | 19.2593 | 15.0922 | 125.1359 | 111.2952 | 100.5385 |
| $k_4$ | 2.3364 | 2.0024 | 1.7414 | 7.6846 | 7.5203 | 7.3143 | 23.7623 | 16.8049 | 13.2117 | 107.8760 | 96.3113 | 86.9273 |
| $k_5$ | 2.1026 | 1.6169 | 1.6307 | 7.7269 | 7.0094 | 6.8876 | 17.9763 | 14.4084 | 12.0654 | 85.8827 | 79.5819 | 74.4692 |
| $k_{N1}$ | 2.0667 | 1.6063 | 1.6036 | 7.5343 | 6.8763 | 6.7574 | 17.9416 | 14.2413 | 11.8623 | 84.9353 | 78.6360 | 73.4395 |
| $k_{N2}$ | 2.0581 | 1.6090 | 1.5976 | 7.4499 | 6.8610 | 6.7238 | 17.9446 | 14.2226 | 11.8579 | 84.9656 | 78.3751 | 73.1182 |
| $k_{N3}$ | 2.1998 | 1.6988 | 1.7116 | 8.1117 | 7.4657 | 7.2623 | 18.5642 | 15.0726 | 12.8917 | 90.3647 | 82.3965 | 77.3164 |
| $k_{N4}$ | 2.3118 | 1.8764 | 1.7125 | 7.9974 | 7.6855 | 7.4613 | 22.2105 | 16.6246 | 13.3351 | 101.7014 | 92.7855 | 85.0792 |
| OLS | 4.2967 | 3.1334 | 2.9932 | 15.5416 | 13.8163 | 13.1563 | 38.8491 | 28.8404 | 23.3809 | 185.4425 | 159.1808 | 145.3175 |
| $\rho = 0.85, p = 8$ | | | | | | | | | | | | |
| $n$ | 20 | 50 | 100 | 20 | 50 | 100 | 20 | 50 | 100 | 20 | 50 | 100 |
| $k_1$ | 3.1444 | 2.5682 | 2.4620 | 21.4056 | 17.7387 | 10.8207 | 54.3085 | 36.0596 | 25.1449 | 202.0053 | 156.8762 | 119.7675 |
| $k_2$ | 3.3571 | 2.6303 | 2.5101 | 28.3796 | 22.1753 | 12.2457 | 75.8952 | 47.4561 | 30.9778 | 284.6723 | 211.9725 | 154.2272 |
| $k_3$ | 3.8979 | 2.9825 | 2.7762 | 29.1936 | 22.5715 | 12.8455 | 72.4889 | 46.6600 | 31.2573 | 275.8329 | 201.8615 | 150.6953 |
| $k_4$ | 3.3834 | 2.6130 | 2.4645 | 25.8376 | 19.3626 | 11.0033 | 59.1447 | 39.2020 | 26.5278 | 238.7088 | 172.1409 | 128.3152 |
| $k_5$ | 2.9414 | 2.4781 | 2.3925 | 17.5096 | 15.6459 | 10.2494 | 47.5644 | 31.5818 | 22.8063 | 170.9078 | 139.3364 | 108.1431 |
| $k_{N1}$ | 2.8966 | 2.4293 | 2.3448 | 17.6051 | 15.5167 | 10.0590 | 46.9861 | 31.2769 | 22.5121 | 170.3857 | 137.6626 | 106.6644 |
| $k_{N2}$ | 2.8683 | 2.3926 | 2.3049 | 17.6876 | 15.4309 | 9.9242 | 46.4738 | 31.0563 | 22.3054 | 170.4134 | 136.3521 | 105.6248 |
| $k_{N3}$ | 2.9750 | 2.5064 | 2.4100 | 17.4701 | 15.6132 | 10.4023 | 46.9108 | 31.4888 | 22.9707 | 171.0634 | 139.1297 | 109.0800 |
| $k_{N4}$ | 3.3558 | 2.6441 | 2.4915 | 23.5494 | 19.0878 | 11.3337 | 57.9280 | 38.4865 | 26.5494 | 219.3679 | 165.6201 | 125.2557 |
| OLS | 6.0621 | 4.6611 | 4.3699 | 39.0492 | 31.7765 | 19.6167 | 101.3570 | 63.3548 | 44.6007 | 375.4153 | 279.4298 | 209.6424 |

(*continued on next page*)

**Table 3** (*continued*)

| $\sigma^2$ | 0.1 | | | 0.5 | | | 1.0 | | | 5.0 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho = 0.95$, $p = 8$ | | | | | | | | | | | | |
| $n$ | 20 | 50 | 100 | 20 | 50 | 100 | 20 | 50 | 100 | 20 | 50 | 100 |
| $k_1$ | 10.1517 | 8.3145 | 7.0777 | 62.3589 | 43.5980 | 38.5160 | 256.3495 | 114.5677 | 72.2927 | 510.1337 | 443.1300 | 441.6436 |
| $k_2$ | 12.4626 | 10.0550 | 8.1842 | 91.6400 | 61.7180 | 53.2412 | 394.4763 | 175.5814 | 104.6281 | 797.0015 | 679.2201 | 677.2535 |
| $k_3$ | 12.9914 | 10.5799 | 8.6537 | 84.9505 | 55.2194 | 50.1042 | 367.1545 | 150.6510 | 92.8478 | 668.6289 | 586.6407 | 598.2785 |
| $k_4$ | 10.7853 | 9.1630 | 7.4245 | 69.2146 | 44.3726 | 41.7137 | 320.9709 | 122.6003 | 76.8874 | 554.7026 | 497.4623 | 516.3498 |
| $k_5$ | 9.3234 | 7.4015 | 6.6582 | 53.1680 | 40.5554 | 33.5896 | 195.5032 | 103.2034 | 65.7234 | 464.0262 | 388.1869 | 376.9323 |
| $k_{N1}$ | 9.1688 | 7.3301 | 6.5494 | 52.6306 | 39.7718 | 33.2534 | 196.4644 | 101.5887 | 64.7785 | 455.3866 | 383.3861 | 374.3269 |
| $k_{N2}$ | 8.9755 | 7.2614 | 6.4089 | 52.1332 | 38.6766 | 32.9210 | 199.8805 | 99.4564 | 63.6095 | 443.7877 | 378.0306 | 372.6178 |
| $k_{N3}$ | 8.9577 | 7.2644 | 6.4152 | 52.1043 | 38.5061 | 32.9412 | 201.9508 | 98.9504 | 63.6302 | 441.7225 | 377.8803 | 372.6184 |
| $k_{N4}$ | 10.9554 | 9.0699 | 7.5963 | 67.0785 | 45.7474 | 41.3113 | 283.3606 | 121.9183 | 76.9880 | 534.8243 | 472.5697 | 481.2463 |
| OLS | 19.5364 | 14.9055 | 12.6329 | 116.0329 | 78.4528 | 66.3018 | 453.3480 | 201.7302 | 127.7137 | 961.3745 | 780.6791 | 758.4431 |

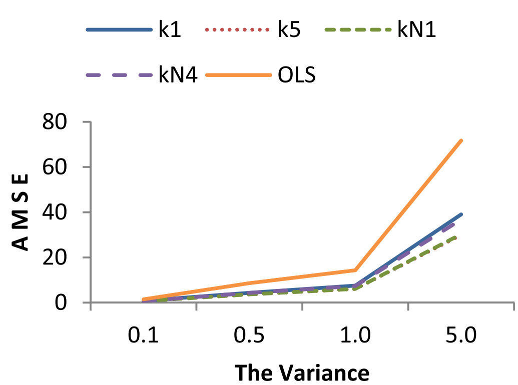


**Figure 1** $n = 20$, $p = 4$, $\rho = 0.75$.

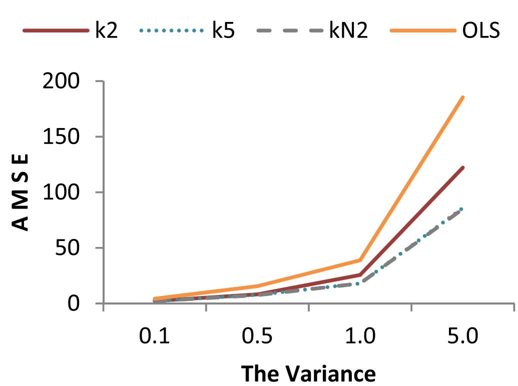


**Figure 2** $n = 20$, $p = 8$, $\rho = 0.75$.

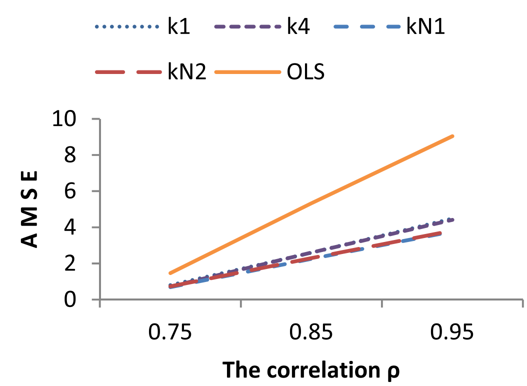


**Figure 3** $n = 20$, $p = 4$, $\sigma^2 = 0.1$.

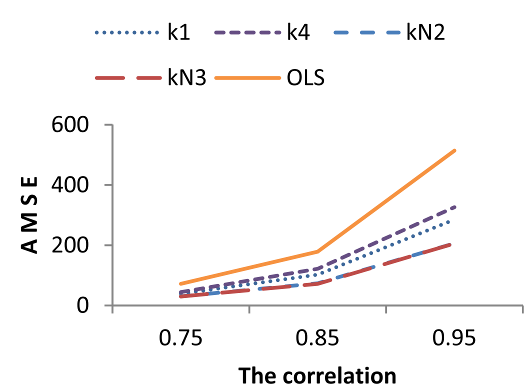


**Figure 4** $n = 20$, $p = 4$, $\sigma^2 = 5.0$.

*4.2.1. Comparison according to the variance $\sigma^2$*

In the previous section, we see that an increase in the variance of the errors $\sigma^2$ makes an increase in the mean squared error. Similarly, there is an increase in biases when we increase the variances of errors. $k_1$, $k_5$, $k_{N1}$ and $k_{N3}$ are the selected estimators to be compared. For given cases $n = 20$, $p = 4$, $\rho = 0.75$ and $n = 20$, $p = 8$, $\rho = 0.75$, biases of the selected estimators are given in Figs. 7 and 8 respectively.

From these figures, we see that increasing the variance gives an increase in biases. $k_{N1}$ has a better performance i.e. it has a small bias among the estimators $k_1$, $k_5$, $k_{N1}$ and $k_{N3}$ for the given cases. It is obvious that if we increase the number of variable $p$ from 4 to 8, then there is a small increase in the bias values fixing $n$ and $\rho$. This is valid for all similar cases.

*4.2.2. Comparison according to the correlation $\rho$*

When the correlation $\rho$ increases, biases of estimators increase. In most cases, the estimators $k_{N1}$ and $k_{N3}$ have the least biases. Especially when $p = 8$, $k_5$ is better than $k_{N3}$ but $k_{N1}$ is again the best estimator. For given values $n = 20$, $p = 4$, $\sigma^2 = 0.1$

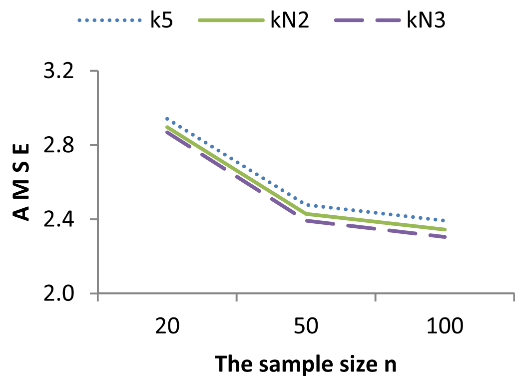


**Figure 5** $p = 8$, $\rho = 0.85$, $\sigma^2 = 0.1$.

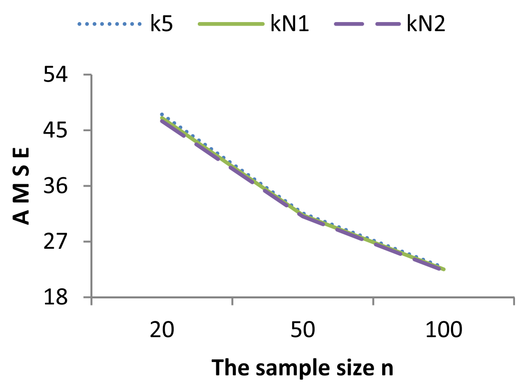


**Figure 6** $p = 8$, $\rho = 0.85$, $\sigma^2 = 1.0$.

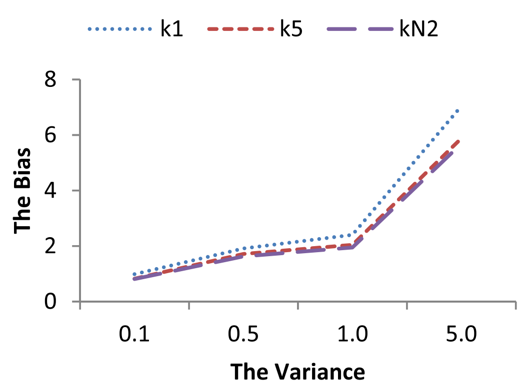


**Figure 7** $n = 20$, $p = 4$, $\rho = 0.85$.

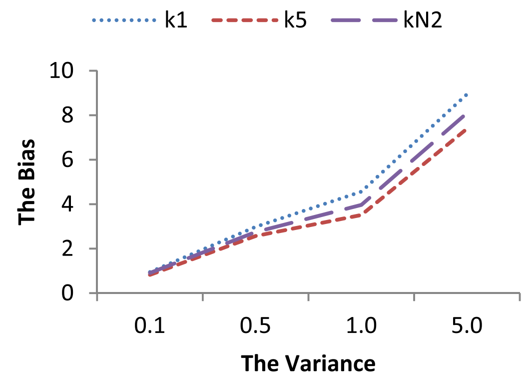


**Figure 8** $n = 20$, $p = 8$, $\rho = 0.85$.

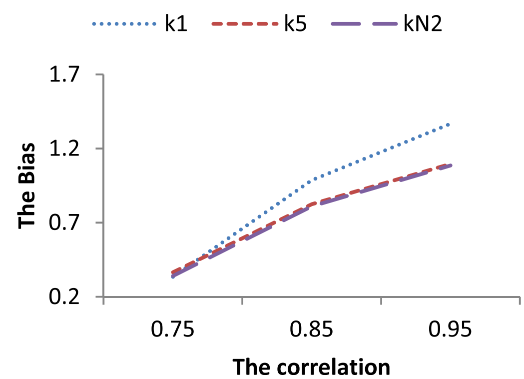


**Figure 9** $n = 20$, $p = 4$, $\sigma^2 = 0.1$.

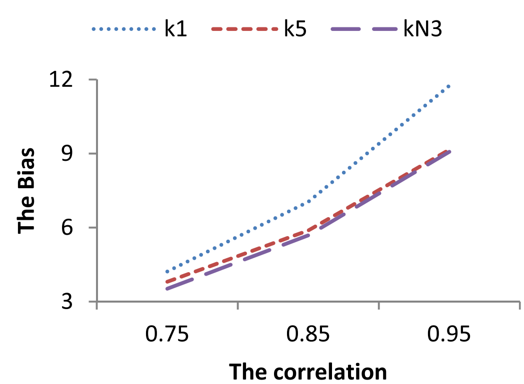


**Figure 10** $n = 20$, $p = 4$, $\sigma^2 = 5.0$.

and $n = 20$, $p = 4$, $\sigma^2 = 5$, biases of the estimators are provided in Figs. 9 and 10. It is obvious from these figures that having a small variance namely $\sigma^2 = 0.1$, an increase in the correlation gives a small increase in biases. However the increase in biases is larger when $\sigma^2 = 5$, approximately ten times larger than the former case.

*4.2.3. Comparison according to the sample size n*

From previous sections, we see that the AMSE decreases as the sample size $n$ increases. Similarly, biases of the selected estimators decrease as the sample size increases. For any combination of $\rho$, $\sigma^2$ and $p$, we observe that the bias decreases as $n$ increases. For given values $p = 4$, $\rho = 0.95$, $\sigma^2 = 0.1$ and $p = 4$, $\rho = 0.95$, $\sigma^2 = 5$, we have given the following Figs. 11 and 12. According to these figures, $k_{N1}$ and $k_{N3}$ have better performances than the other estimators. In most of the situations $k_{N1}$ has the least bias. In some cases, $k_5$ is better than $k_{N3}$ especially when $p = 8$, but it does not have a better performance than $k_{N1}$.

## 5. A real data application

To illustrate the findings of the paper, real life data have been analyzed in this section. The data are obtained from official

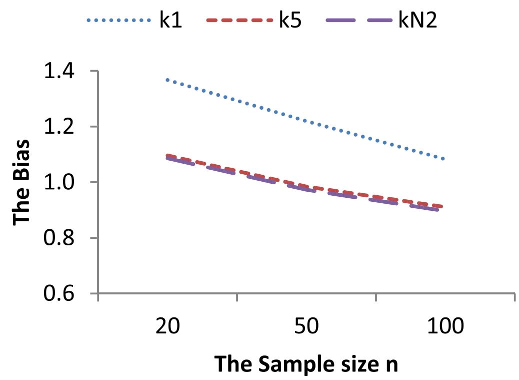


**Figure 11** $p = 4$, $\rho = 0.95$, $\sigma^2 = 0.1$.

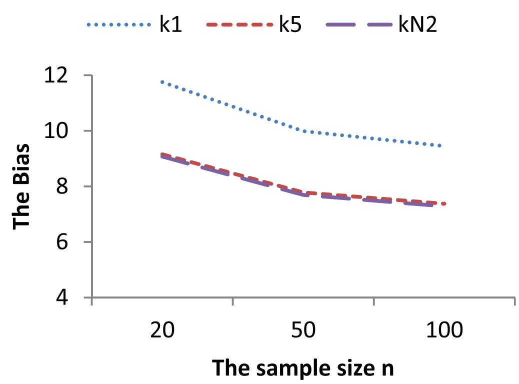


**Figure 12** $p = 8$, $\rho = 0.85$, $\sigma^2 = 5.0$.

web site of the Turkish Statistical Institute (see http://www.turkstat.gov.tr/). The characteristic of the data is as follows:

Wide lands and underground sources are not the only wealth of countries. Some indicators are calculated to put forth the exact power of countries. In order to compare the wealth of nations, one of the most important indicators is the Gross Domestic Products (GDP). We have modeled GDP by cost components (at 1987 and 1998 prices) between the years 1968 and 2008, closely concerning the economy of Turkey in parallel to the growth and trends in the world. We have explained GDP using some parameters in the axes of foreign trade and production by multiple linear regression.

The dependent variable is the GDP of Turkey. The eight explanatory variables are the following respectively: $X_1$: export, $X_2$: import, $X_3$: energy production, $X_4$: number of establishments in manufacturing industry, $X_5$: number of employees in manufacturing industry, $X_6$: wheat production, $X_7$: milk production and finally $X_8$: meat production.

We have seen that the model has a multicollinearity problem since the condition number is $\kappa = \sqrt{\frac{\lambda_{\max}}{\lambda_{min}}} = 49.1128 > 30$ which shows severe multicollinearity. We have given the correlation matrix of the GDP data in Table 4. One can see from that table that there are high correlations among the explanatory variables. The MSE values of the given estimators are provided in Table 5. It can be seen from Table 5 that $k_{N1}$, $k_{N2}$ and $k_{N3}$ have less MSE than the others, especially $k_{N2}$ which has the best performance in the sense of MSE. Moreover, if we look at the determination coefficients and PRESS statistics of each model, it can be said that using these biased estimators makes no significant change in the model predictability. Thus, we advise to use the new defined estimators rather than the others.

**Table 5** MSE values of the estimators in the application.

| $k$ | $k$ values | MSE | Variance | Sq. Bias | $R^2$ | PRESS |
|---|---|---|---|---|---|---|
| $k_1$ | 0.0022 | 0.1551 | 0.0804 | 0.0747 | 0.9880 | 0.0137 |
| $k_2$ | 0.0023 | 0.1564 | 0.0785 | 0.0780 | 0.9879 | 0.0137 |
| $k_3$ | 0.0098 | 0.2726 | 0.0230 | 0.2496 | 0.9816 | 0.0146 |
| $k_4$ | 0.0026 | 0.1615 | 0.0723 | 0.0892 | 0.9876 | 0.0137 |
| $k_5$ | 0.0008 | 0.1495 | 0.1312 | 0.0184 | 0.9895 | 0.0134 |
| $k_{N1}$ | 0.0009 | 0.1480 | 0.1262 | 0.0219 | 0.9894 | 0.0135 |
| $k_{N2}$ | 0.0011 | 0.1464 | 0.1178 | 0.0285 | 0.9892 | 0.0135 |
| $k_{N3}$ | 0.0010 | 0.1471 | 0.1224 | 0.0247 | 0.9893 | 0.0135 |
| $k_{N4}$ | 0.0018 | 0.1498 | 0.0902 | 0.0596 | 0.9884 | 0.0136 |
| OLS | 0.0000 | 0.1916 | 0.1916 | 0.0000 | 0.9906 | 0.0134 |

## 6. Conclusion

In this paper, we reviewed some new modified ridge parameters and the ones proposed earlier. At first, we explained the multicollinearity and gave necessary information about methodology of the ridge regression. We introduced, secondly, ten ridge estimators half of which were proposed earlier and the other half were our proposals. Then, we compared the parameters according to their performance evaluating the average mean squared errors and also biases. The simulation study was performed for different combinations of the variances of the error terms ($\sigma^2$), the numbers of explanatory variables ($p$), the numbers of observations ($n$) and different correlation coefficients between the predictors ($\rho$). We found that our proposals are better than the ones proposed by $k_1$: Hoerl and Kennard (1970a), $k_2$: Lawless and Wang (1976), $k_3$: Kibria

**Table 4** The correlation matrix of the GDP data.

| $X_i$ | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ | $X_7$ | $X_8$ |
|---|---|---|---|---|---|---|---|---|
| $X_1$ | 1.0000 | 0.9971 | 0.8961 | 0.9063 | 0.9456 | 0.4113 | 0.6804 | 0.4468 |
| $X_2$ | 0.9971 | 1.0000 | 0.8978 | 0.8977 | 0.9399 | 0.4201 | 0.6793 | 0.4437 |
| $X_3$ | 0.8961 | 0.8978 | 1.0000 | 0.8199 | 0.8902 | 0.5373 | 0.7435 | 0.4946 |
| $X_4$ | 0.9063 | 0.8977 | 0.8199 | 1.0000 | 0.9643 | 0.3249 | 0.5092 | 0.2572 |
| $X_5$ | 0.9456 | 0.9399 | 0.8902 | 0.9643 | 1.0000 | 0.5201 | 0.6869 | 0.4622 |
| $X_6$ | 0.4113 | 0.4201 | 0.5373 | 0.3249 | 0.5201 | 1.0000 | 0.7583 | 0.6950 |
| $X_7$ | 0.6804 | 0.6793 | 0.7435 | 0.5092 | 0.6869 | 0.7583 | 1.0000 | 0.8756 |
| $X_8$ | 0.4468 | 0.4437 | 0.4946 | 0.2572 | 0.4622 | 0.6950 | 0.8756 | 1.0000 |

(2003), $k_4$ and $k_5$: Dorugade (2014) according to their AMSE and bias performance. Finally, we conclude that our estimators are satisfactory over the multicollinearity problem and among our estimators $k_{N4}$ has the best performance. The estimator $k_{N1}$ has the least bias in most situations. However dealing with real data, the case may differ. Therefore we highly recommend researchers not to use just one ridge estimator to overcome their problem and not to decide without further study.

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